Static Deformation Due to Shear and Tensile Faults in a Layered Half-Space

by Yu-Mei He, Wei-Min Wang, and Zhen-Xing Yao

Abstract Based on the generalized reflection and transmission coefficient matrix method, formulations for surface static displacements in a layered half-space are extended to include tensile and inflation point sources from a point pure shear dislocation source. Equations for calculating internal displacement fields from these sources are also derived. The validity of the formula and precision of the new method are illustrated by comparing the consistency of our results and the analytical solutions given by Okada’s (1985, 1992) code in a homogenous half-space and Wang et al.’s (2003) numerical solutions in a multilayered half-space. We also study the effect of a layered half-space on the surface displacement created by various finite faults. Several typical velocity structures in reality are selected. For strike-slip, reverse dip-slip, and tensile finite-fault models, the focal depth is very sensitive to the presence of the layered model. The slip displacement is more sensitive to the layered model in the case of the normal dip-slip sources. More numerical tests show that the sensitive slip is mainly due to the ultralow-velocity topsoil. For inflations, the source depth and volume change also altered due to the layered model.

Introduction

Static deformation data can be used to investigate geological fault movements and related stress distributions. Maruyama (1964) gave the analytical solution of a static elastic dislocation in an infinite or semi-infinite media. The analytical solutions for surface and internal deformations generated by shear and tensile faults in a half-space were derived by Okada (1985, 1992) and were widely used. However, a homogeneous half-space model may oversimplify the real Earth. For example, the existence of a soft top layer tends to generate surface displacements that are larger near the epicentral area but decrease rapidly with distance (Sato and Matsu’ura, 1973). Numerical tests also revealed that, for a pure shear thrust fault with a 30° dip angle, the response generated using a homogeneous half-space model can underestimate the displacement by up to 10% if the observation point is located in the hanging wall, and it will overestimate the displacement by up to 30% if the observation point is located in the footwall (Ji et al., 2001). Therefore, when dealing with geodetic data, a more realistic layered Earth model is often preferred (Hearn et al., 2002).


In this article, following Xie and Yao (1989), we expand their approach to include tensile and inflation sources. Formulations for both surface and internal displacement fields are derived. Numerical tests are conducted to examine these equations. To validate our method, we compare our point source solutions to Okada’s (1985) analytic results and Wang et al.’s (2003) numerical solutions. Using the new method, the responses for various finite faults buried in different layered models are calculated and compared to Okada’s rectangular fault solutions in a homogenous half-space. The effects of focal depth, slip, dip, and rake angles are investigated.

Theory

In the frequency domain, the homogeneous elastostatic equation (without the source term) can be derived from the
elastodynamic equation by making the circular frequency
\( \omega \to 0 \) (Zhu and Rivera, 2002). Since the Lamé theorem
remains true even for static problems, displacement poten-
tials can be adopted and the vector elastostatic equation can
be turned into simpler scalar Laplacian equations. For an
isotropic layered half-space, the solutions of displacement
potentials are similar to those in the elastic dynamic prob-
lem. It is convenient to derive source functions following
methods used in wave motion problems (Yao and Harkrider,
1983). Although there is no wave propagation in a static
problem, the existence of an interface still distorts static dis-
placement fields on both sides of it. We will borrow ideas
of reflection and transmission from the wave problem but
rename them as static reflection and transmission. Similarly,
without frequency dependency, terms related to \( \exp(-kz) \)
and \( \exp(+kz) \) are no longer linked with up- and downgoing
waves. They are actually different eigenfunctions represent-
ning static displacement fields. However, for the ease of de-
scription, we sometimes still call them up- and downgoing
disturbances of static displacements. Following these pro-
cesses, the original generalized reflection and transmission
coefficient matrix method for the wave propagation problem
can be modified for dealing with the static problem.

Tensile and Inflation Source Functions

Xie and Yao (1989) derived source terms for static shear
dislocations. Similarly, we can obtain source functions and
related source coefficients for an arbitrarily oriented tensile
dislocation.

According to Steketeer (1958), the displacement field
\( u_i(x_1, x_2, x_3) \) due to a dislocation \( \Delta u_j(x_1, x_2, x_3) \) across a
surface \( S \) in an isotropic medium is given by

\[
\begin{align*}
  u_i &= \int_S \Delta u_j \left[ i\delta_{jk} \frac{\partial u^m}{\partial x_n} + \mu \left( \frac{\partial u^n_{ij}}{\partial x_k} + \frac{\partial u^m_{ij}}{\partial x_n} \right) \right] n_k dS, \\
  & \quad \text{(1)}
\end{align*}
\]

where \( \delta_{jk} \) is the Kronecker delta, \( i \) and \( \mu \) are Lamé constants,
\( n_k \) is the direction cosine of the normal to the surface element
d\( S \), and \( u^m_{ij} \) is the \( i \)th component of the displacement at \( (x_1,
\quad x_2, x_3, \quad) \) due to a point force at \( (\xi_1, \quad \xi_2, \quad \xi_3) \) acting in the \( j \)th
direction.

For a fault with dip \( \delta \) and strike \( \theta \) and buried in an
infinite space (shown in Fig. 1), a tensile dislocation can be
expressed as \( U_3y \). Here \( y \) is the direction of the rupture vec-
tor of the fault plane, and \( U_3 \) is the scalar displacement
perpendicular to the fault. Then, the displacement potential
functions \( \phi, \psi, \) and \( \chi \) of a static tensile point source can be written as

\[
\begin{align*}
  \phi &= \frac{S}{4\pi} \sum_{m=0}^{2} U_3A_m \int_{-\infty}^{\infty} P_m e^{-kz-h} J_m(kr) dk d, \\
  \psi &= \frac{S}{4\pi} \sum_{m=0}^{2} U_3A_m \int_{-\infty}^{\infty} SV_m e^{-kz-h} J_m(kr) dk d, \\
  \chi &= \frac{S}{4\pi} \sum_{m=0}^{2} U_3A_m + 3 \int_{-\infty}^{\infty} SH_m e^{-kz-h} J_m(kr) dk d,
\end{align*}
\]

where \( S \) is the area of the fault plane, \( h \) is the \( z \) coordinate of
the point source, \( J_m(kr) \) is the ordinary Bessel function of
order \( m \), and

\[
\begin{align*}
  A_0 P_0 &= A_0^{(1)} P_0^{(1)} + A_0^{(2)} P_0^{(2)}, \\
  A_0 SV_0 &= A_0^{(1)} SV_0^{(1)} + A_0^{(2)} SV_0^{(2)}, \\
  A_0^{(1)} &= 1, \\
  A_0^{(2)} &= -\sin^2 \delta, \\
  A_1 &= \sin(2\delta) \sin \theta, \\
  A_2 &= -\sin^2 \delta \cos(2\theta), \\
  A_3 &= -\sin(2\delta) \cos \theta, \\
  A_4 &= \sin^2 \delta \cos(2\theta)
\end{align*}
\]

are orientation factors. \( P_m, SV_m, \) and \( SH_m \) are source coef-
ficients and can be expressed as

\[
\begin{align*}
  P_0^{(1)} &= -\frac{\Delta}{1+\Delta}, \\
  P_0^{(2)} &= \frac{1}{2(1+\Delta)}, \\
  P_1 &= -\frac{\varepsilon \Delta}{1+\Delta}, \\
  P_2 &= \frac{1}{2(1+\Delta)}, \\
  SV_0^{(1)} &= -\frac{3}{2(1+\Delta)}, \\
  SV_0^{(2)} &= \frac{\varepsilon}{1+\Delta}, \\
  SH_1 &= \varepsilon, \\
  SH_2 &= -1,
\end{align*}
\]

where \( P_m, SV_m, \) and \( SH_m \) represent \( P_m^+, \quad P_m^-, \quad SV_m^-, \quad SV_m^- \)
and \( SH_m^+, \quad SH_m^- \), plus and minus superscripts denote up-
and downgoing static disturbances, \( \varepsilon \) is \( 1 \) for the minus
superscript and equal to \( 1 \) for the plus superscript, and
\( \Delta = (\alpha + \mu)(\lambda + 3\mu) \).

Compared with static point shear dislocation source
functions (Xie and Yao, 1989), we have the same source
coefficients except for \( P_0 \) and \( SV_0 \). For a tensile dislocation,
its rupture vector parallels the direction of the fault-plane normal, and its rake \( \lambda \) disappears, which means orientation factors for shear and tensile sources are different.

For the inflation point source, we have a simple expression that

\[
\varphi = \frac{V}{4\pi} \int_0^\infty P_0 e^{-ikr} J_0 (kr) dk,
\]

(5)

where \( V \) is the volume change, and

\[
P_0 = \frac{\Delta - 1}{\Delta + 1}.
\]

Static Displacement Solutions for Shear, Tensile, and Inflation Sources in a Layered Half-Space

Consider \( n - 1 \) parallel, homogeneous, and isotropic layers overlaying a semi-infinite medium (Fig. 2). A right-handed cylindrical coordinate system is used, and the \( z \) axis is vertically downward. The layers and interfaces are labeled according to their distances away from the free surface.

Following the generalized reflection and transmission coefficient matrix method (shortened to generalized R/T coefficient matrix method [Kennett, 1983]), using the static shear dislocation source given by Xie and Yao (1989) and tensile source derived in the last section, the surface and internal displacement fields can be expressed as

\[
w_z = \frac{S}{4\pi} \sum_{j=1}^{3} \sum_{m=0}^{3} U_j A_m \int_0^\infty w_m J_m (kr) dk,
\]

\[
q_r = \frac{S}{4\pi} \sum_{j=1}^{3} \sum_{m=0}^{3} U_j A_m \int_0^\infty q_m J_m (kr) - \frac{m}{kr} J_m (kr) dk,
\]

(7)

\[
v_o = \frac{S}{4\pi} \sum_{j=1}^{3} \sum_{m=0}^{3} U_j A_m + I \int_0^\infty [q_m \frac{m}{kr} J_m (kr) - \frac{m}{kr} J_m (kr)] dk,
\]

where \( U_1, U_2, \) and \( U_3 \) correspond to the strike-slip, dip-slip, and tensile components. The \( w_m, q_m, \) and \( v_m, m = 1, 2, 3 \) represent the displacement fields under vector surface harmonic base, and their detailed expressions will be given later. The orientation factors are

\[
A_{10} = 0,
A_{11} = \cos \delta \cos \theta,
A_{12} = \sin \delta \sin (2\theta),
A_{14} = -\cos \delta \sin \theta,
A_{15} = \sin \delta \cos (2\theta),
A_{16} = 1,
A_{20} = \sin \delta \cos \delta,
A_{21} = -\cos (2\delta) \sin \theta,
A_{22} = \sin \delta \cos \delta \cos (2\theta),
A_{24} = -\cos (2\delta) \cos \theta,
A_{25} = -\sin \delta \cos \delta \sin (2\theta),
A_{30} = \cos \delta \cos \theta,
A_{31} = \sin \delta \sin (2\theta),
A_{32} = \sin \delta \cos (2\theta),
A_{33} = \sin ^2 \delta \cos (2\theta),
A_{34} = -\sin \delta \cos (2\theta),
A_{35} = \sin ^2 \delta \cos (2\theta).
\]

\[
(9)
\]

For calculating the surface displacement, \( w_m, q_m, \) and \( v_m \) can be expressed as

\[
(10)
\]

\[
(11)
\]

Then the surface displacements are obtained by putting equations (10) and (11) into equations (7) and (9). Here \( I \) is a unit matrix, the subscript \( L \) refers to the \( SH \) problem. Source coefficients \( P_m, SV_m, \) and \( SH_m \) are given by equation (4). For the \( P-SV \) problem, (i.e., equation 10),

\[
\begin{bmatrix}
0 \\
1 / \Delta_1
\end{bmatrix}
\]

(12)
is the static reflection coefficient matrix at the free surface, and

\[ R_{ev} = (1 + \Delta)(1/\Delta - 1) \]  

is the receiver function matrix. Their detailed expressions are given in the Appendix. \( R_{DS}^{\text{FS}} \) is the upgoing static generalized reflection coefficient matrix between the free surface \( z_1 \) and source depth \( z_m^0 \). \( R_{R}^{\text{FS}} \) and \( T_{R}^{\text{FS}} \) are down- and upgoing static generalized R/T coefficient matrices between \( z_1^0 \) and \( z_m^0 \), and \( R_{DS}^{\text{FS}} \) is the downgoing static generalized reflection coefficient matrix between \( z_1^0 \) and \( z_m^0 \) (see Fig. 2). Detailed expressions of these static R/T coefficient matrices at interfaces are also given in the appendix. For the \( SH \) problem, the static receiver function and reflection coefficient at the free surface are 2 and 1, respectively.

Similar to the wave motion problem (Kennett, 1983), the surface static solution (equations 10 and 11) can be extended to include internal displacement fields. For receivers above the focal depth, that is, \( z_R < z_S \), we have

\[
\begin{align*}
\begin{bmatrix} q_m \\ w_m \end{bmatrix} &= R_{ev}(I - R_{DS}^{\text{FS}} R_{FR}^{\text{FS}})^{-1} T_{DS}^{\text{FS}} (I - R_{DS}^{\text{FS}} R_{FS}^{\text{FS}})^{-1} \\
&+ \begin{bmatrix} R_{DS}^{\text{FS}} (P_m^+ / S_m^0) + (P_m^- / S_m^-) \\ R_{DS}^{\text{FS}} (P_m^+ / S_m^0) + (P_m^- / S_m^-) \end{bmatrix},
\end{align*}
\]

\[ v_m = R_{ev,l}(I - R_{DS}^{\text{UL}} R_{FR}^{\text{UL}})^{-1} T_{DS}^{\text{UL}} (I - R_{DS}^{\text{UL}} R_{FS}^{\text{UL}})^{-1} (R_{DS}^{\text{UL}} S_{m}^+ + S_{m}^-), \]

where

\[ R_{ev} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} R_{FR}^{\text{UL}} \]

are static receiver functions at \( z_R \). \( R_{FR}^{\text{UL}} \) is the upgoing generalized reflection coefficient matrix between the free surface \( z_1 \) and source depth \( z_m^0 \). \( R_{DS}^{\text{UL}} \) and \( T_{DS}^{\text{UL}} \) are the down- and upgoing static generalized R/T coefficient matrices between \( z_1^0 \) and \( z_S^0 \). Apparently, \( z_R = 0 \) makes equations (14) and (15) degenerate to the surface displacement equations (10) and (11).

For receivers below the focal depth (i.e., \( z_R > z_S \), we have

\[
\begin{align*}
\begin{bmatrix} q_m \\ w_m \end{bmatrix} &= R_{ev}(I - R_{DS}^{\text{UL}} R_{RU}^{\text{UL}})^{-1} T_{DS}^{\text{UL}} (I - R_{DS}^{\text{UL}} R_{FS}^{\text{UL}})^{-1} \\
&+ \begin{bmatrix} R_{DS}^{\text{UL}} (P_m^+ / S_m^0) + (P_m^- / S_m^-) \\ R_{DS}^{\text{UL}} (P_m^+ / S_m^0) + (P_m^- / S_m^-) \end{bmatrix}.
\end{align*}
\]

\[ v_m = R_{ev,l}(I - R_{DS}^{\text{UL}} R_{RU}^{\text{UL}})^{-1} T_{DS}^{\text{UL}} (I - R_{DS}^{\text{UL}} R_{FS}^{\text{UL}})^{-1} (-R_{DS}^{\text{UL}} S_{m}^+ + S_{m}^-), \]

where

\[ R_{ev} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} R_{RU}^{\text{UL}} \]

are static receiver functions for \( P-SV \) and \( SH \) problems at \( z_R \), respectively. \( R_{DS}^{\text{UL}} \) and \( T_{DS}^{\text{UL}} \) are down- and upgoing static generalized R/T coefficient matrices between \( z_1^0 \) and \( z_S^0 \). Substituting equations (14) and (15) or (18) and (19) into equations (7) and (9), we can calculate internal displacement fields in a layered model.

**Numerical Examples**

After the derivation of the source terms for tensile and inflation sources, we obtain the solution of the surface static displacements in a layered half-space due to shear, tensile, and inflation point sources. As numerical examples, we investigate three aspects. At first, we test the accuracy of the formulations by comparing our solutions with Okada’s (1985) analytical results in a homogeneous half-space and Wang et al.’s (2003) numerical solutions in a multilayered half-space. Secondly, because in most cases we need to calculate the response of finite faulting using a point source approximation, we will discuss the size of the proper sub-fault area that can be substituted by a point source with various focal depths. Finally, because a uniform half-space Earth model is usually used in inverting geodetic data to estimate the coseismic source mechanism, we are generally concerned how a homogenous half-space model can bias the estimation of fault parameters compared to a layered half-space model. Here we make a system analysis on the effect of various medium models and source geometries on parameters estimated by static deformation data.

To test the accuracy of equations derived in the last section, the static responses of a point source in a homogenous half-space are calculated using our method and compared with Okada’s analytical solutions in Cartesian coordinates. In these numerical tests strike-slip, dip-slip, tensile sources with various dip angles as well as inflation sources are used. Their focal depths are fixed at 5 km. We indicate the discrepancy between our results with Okada’s analytical solutions using the relative error value defined as

\[
\frac{\delta u}{u} = \frac{u_{\text{this paper}} - u_{\text{okada}}}{u_{\text{okada}}}, \quad u_{\text{okada}} \neq 0.
\]

The relative error values of results with different point
source models are displayed in Figure 3, in which the open circles indicate relative error values and the crosses are the vertical displacements from Okada’s solutions. Generally, \( u_{\text{this paper}} \) would reach to \( \pm 10^{-7} \) cm when \( u_{\text{Okada}} \) equals 0. We also calculate the relative error values of vertical and horizontal displacements adopting various models with various focal depths, dip, and rake angles at different observation points \((x, y)\). In these numerical tests, the maximum discrepancy is less than 2% of the displacement \( u \) generally. Furthermore, we compare our point source solutions with Wang et al.’s numerical results in a multilayered half-space. The velocity structure of the layered half-space model is given in Table 1. The discrepancy between our results and Wang et al.’s is presented in Figure 4. Strike-slip and dip-slip sources with various dip angles are used. Their focal depths are 5 km. From the figure, we can see that larger discrepancies mainly correspond with lower displacements \( u \) and the maximum discrepancy is less than 5% of the displacement usually. The two kinds of comparisons validate our method.

Since static deformations decay rapidly with distance from the source, ground deformation modeling is made ordinarily near the fault, so a point source approximation is almost never valid (Lay and Wallace, 1995). In our method we divide the finite fault into several subfaults that are substituted by the point sources. Beyond all doubt, the subfault would be a point source when its area decreases infinitely. However, utilization of the very small area of subfaults will require abundant computer time. The proper size of subfaults is a trade-off between precision and efficiency. In the fol-

Table 1

<table>
<thead>
<tr>
<th>( V_p ) (km/sec)</th>
<th>( V_s ) (km/sec)</th>
<th>( \rho ) (g/cm³)</th>
<th>( Th ) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>1.15</td>
<td>1.37</td>
<td>1.00</td>
</tr>
<tr>
<td>6.20</td>
<td>3.58</td>
<td>3.00</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

\( V_p, V_s, \rho, \text{ and } Th \) are the P-wave velocity, S-wave velocity, density, and thickness of each layer, respectively. The values of model-layer rigidity are derived from the velocity models.
The development of this article is the derivation of the source terms for tensile and inflation point sources, as well as the use of a layered half-space model. We will now discuss how a homogenous half-space model can bias the estimation of fault parameters than a layered half-space model. In the numerical experiments we calculate the static displacement as “data” based on a prescribed source in a layered half-space model, then deduce its source parameters by fitting this data based on a homogenous half-space model. In the forward calculation, we set up a 6 km × 4 km planar rectangle fault buried in a layered half-space and divided the finite fault into 24 1 km × 1 km subfaults (see Fig. 6). Each subfault has the same slip dislocation of 1 m. Substituting point source for subfault, we obtain the vertical and horizontal ground displacement data by our method. In the inversion we utilize a hybrid global search algorithm (Liu et al., 1995) and calculate the responses using Okada’s analytical solutions of a planar rectangular fault. In Okada’s code a single 6 km × 4 km fault with one slip displacement is applied. The strike, length, and width of the fault are fixed, and only the focal depth, slip displacement, rake, and dip angles are inverted. Nevertheless, before our investigation of the effect of the layered model, we need to test the efficiency of our forward and inverse methods first. We construct the data using the homogenous half-space model and our method and invert it with Okada’s solutions. Various source mechanisms are investigated. Table 2 shows that in any cases the inverted focal depth, slip displacement, rake, and dip angles are quite close to the real values. All these facts validate the reliability of our forward and inverse method. Hereafter we will probe the effect of medium model.

Several typical fault models and corresponding velocity structure in reality are investigated. The general structures for a strike-slip fault (e.g., San Andreas fault) and reverse dip-slip earthquake (e.g., Chi-Chi earthquake), the typical rift structure (e.g., Corinth Gulf) for normal and tensile faulting earthquakes, and the structure of an active volcano (e.g., Mt. Etna) for inflation sources are selected. The layered earth models of different earthquake zones are given in Tables 3, 4, 5, and 6, respectively. For all kinds of designed fault models, the focal depth, rake, and slip displacement are fixed and the dip angle is changed from 30° to 90°. The final inverse results including focal depth, slip displacement, dip, and rake angles are given in Table 7.

Since the 1989 Loma Prieta earthquake ruptured a part of the San Andreas fault, we choose the Loma Prieta velocity structure model (Table 3; Wald, et al. 1991) to discuss the strike-slip fault. From Table 7 we observe that the inverted rake and dip angles almost match the “true” values of the layered models. The two parameters sensitive to the existence of the layered half-space model are the focal depth and slip displacement. The largest discrepancy with the real value in focal depth and slip can reach 12% and 7%, respectively. For reverse dip-slip model, we select the layered central Taiwan Earth model (Table 4) given by Ma et al.
Figure 5. The discrepancy of maximum vertical displacements created by different finite fault area with that by point source. Various focal depths are investigated and illustrated by different symbols. Here the horizontal axis gives the width of square finite fault, and the vertical axis show the discrepancy.

Figure 6. 3D dislocation model in a layered half-space used to calculate the coseismic displacements with our method.

Table 2
Real and Inverted Results using Homogenous Half-Space Model for 4 by 6 Subfault Plane.

<table>
<thead>
<tr>
<th>Source Type</th>
<th>Focal Depth (km)</th>
<th>Dip (°)</th>
<th>Rake (°)</th>
<th>Slip (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Real 5.00</td>
<td>50.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Inverted 5.28</td>
<td>51.98</td>
<td>0.04</td>
<td>1.05</td>
</tr>
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<td></td>
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<td>70.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Inverted 5.05</td>
<td>70.05</td>
<td>0.02</td>
<td>1.01</td>
</tr>
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<td></td>
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<td>0.00</td>
<td>1.00</td>
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<tr>
<td></td>
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<td>90.00</td>
<td>1.00</td>
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<td></td>
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<td>90.05</td>
<td>1.00</td>
</tr>
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<td>90.00</td>
<td>1.00</td>
</tr>
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<td>50.12</td>
<td>90.52</td>
<td>1.01</td>
</tr>
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<td>1.00</td>
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<td></td>
<td>Inverted 5.01</td>
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<tr>
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<td>1.00</td>
</tr>
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<td></td>
<td>Inverted 4.99</td>
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<td>−89.76</td>
<td>1.00</td>
</tr>
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<td>−90.00</td>
<td>1.00</td>
</tr>
<tr>
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<td>Inverted 4.97</td>
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<td>−90.06</td>
<td>1.00</td>
</tr>
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</tr>
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<td>69.96</td>
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<td>−</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Inverted 4.99</td>
<td>50.05</td>
<td>−</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Real 5.00</td>
<td>70.00</td>
<td>−</td>
<td>1.00</td>
</tr>
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</tr>
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<td></td>
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<td>−</td>
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</tr>
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<td></td>
<td>Inverted 5.04</td>
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<tr>
<td>Inflation</td>
<td>Real 5.00</td>
<td>−</td>
<td>−</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Inverted 4.90</td>
<td>−</td>
<td>−</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The forward calculation used our method. Okada’s (1985) solution is applied to invert focal depth, dip, rake, and slip. For inflation source, only source depth and volume change are investigated.

(1996). The character of the inverse parameters is similar to that of the strike-slip source when the dip angle equals 30°. The largest discrepancy in depth and slip can reach 16% and 4%, respectively. Unexpectedly, the inverted focal depth and slip are almost unchanged when we investigate the reverse dip-slip source with dip angles greater than 50°.

As an active extension regime, the refined velocity structure of Corinth Gulf (Table 5; Zahradnik, 1999) is applied to the normal dip-slip source. Different from the inverted results for strike-slip and reverse dip-slip sources, the results presented in Table 7 show the dip angle is also sensitive to the presence of the layered half-space model as well as focal depth and slip. Moreover, the slip is the most sensitive parameter, and the maximum discrepancy in slip can reach 27%. More numerical tests show that the sensitive slip is mainly due to the ultralow-velocity topsoil. The discrep-
ancy in slip would decrease when the $P$-wave velocity of the topsoil increases (see Table 5 and Table 7: inverted b). The same velocity structure of the Corinth Gulf is also used for the tensile source.

Table 7 shows that the parameter most sensitive to the layered half-space model is the focal depth, and its discrepancy ranges from 18% to 33%. The modified velocity structure of Mt. Etna, Italy (Table 6; Villasenor et al., 1998) is employed for inflation source where rake and dip is not exit and slip displacement is substituted by volume change. The source depth decreases to 4.19 km and the volume changes to 0.91. Another set of inversions is made with only the horizontal displacements since these are often the only components available. The results shown in Table 8 are similar to those in Table 7.

Discussions and Conclusion

Over the last two decades, several workers have investigated the effect of Earth layering on fault displacements for both strike-slip and dip-slip sources (e.g., Sato, 1971; Sato and Matsu’ura, 1973; Savage, 1987; Ma and Kusznir, 1994; Savage, 1998; Cattin et al., 1999). Most of them simply constructed the model of a superficial layer overlying a half-space to represent the layered half-space model. In this article, based on the generalized reflection and transmission coefficient matrix method, formulations for surface static displacements in a layered half-space are extended to include tensile and inflation point sources from a point pure shear dislocation source. We also select several typical velocity structures in reality and various source mechanisms including tensile and inflation and obtain newly quantified results.

Generally speaking, using the layered half-space model, the calculated ground displacements would be greater (less) than that applying homogenous half-space model depending mainly on rigidity increase (decrease) with depth (Ma and Kusznir, 1994). In our article, we choose several velocity structures as approximations of the Earth. Without question the rigidity would increase with depth in these models and the calculated displacements would be amplified. For fitting the higher displacements with a homogenous half-space model, one has to decrease the depth and change the slip and alter the dip angle sometimes. Our results show, for strike-slip sources and reverse dip-slip sources with a dip of 30°, that the focal depth is the main sensitive parameter to the stratified media. The neglect of the layering, especially the low-velocity layers at shallower depth, would cause one to underestimate the depth of the dislocation (Cervelli et al., 2002). Two sets of inversions, the first with the vertical and
slip source with steep dip angle (different source mechanisms. For example, for a reverse dip-
horizontal displacements, have similar results. However, the ef-
tects of the layered half-space model are also varied with
different source mechanisms. For example, for a reverse dip-
slip source with steep dip angle (>50°), the effect of the
layered half-space model is small and can be ignored.

Cattin et al. (1999) analyzed the effect of a layer at
the top of the upper crust on a shallow normal-faulting earth-
quake using analytical and numerical modeling. They con-
cluded that using a homogenous half-space to interpret the
coseismic displacements leads to shallower faults than in
reality in all cases and tends to overestimate the slip by 10%–
20% when horizontal displacements alone are considered. For
normal dip-slip source, we found the ultralow-velocity
topsoil would cause slip much sensitive even though the
vertical and horizontal components are used simultaneously.
When the P-wave velocity of the topsoil increases to a cer-

| Table 7 | Real and Inverted Results using Layered and Homogenous Half-Space Model for 4 by 6 Subfault Plane, Respectively |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Focal Depth | Dip (°) | Rake (°) | Slip (m) |
| (km) | | | |
| Left | Real | 5.00 | 50.00 | 0.00 | 1.00 |
| strike slip | Inverted | 4.64 | 50.12 | –0.16 | 1.04 |
| Real | 5.00 | 70.00 | 0.00 | 1.00 |
| Inverted | 4.40 | 69.51 | 0.00 | 1.06 |
| Real | 5.00 | 90.00 | 0.00 | 1.00 |
| Inverted | 4.44 | 90.23 | –0.02 | 1.07 |
| Reverse | Real | 5.00 | 30.00 | 90.00 | 1.00 |
| dip slip | Inverted | 4.22 | 29.01 | 90.14 | 1.04 |
| Real | 5.00 | 50.00 | 90.00 | 1.00 |
| Inverted | 4.94 | 50.33 | 90.07 | 1.02 |
| Real | 5.00 | 70.00 | 90.00 | 1.00 |
| Inverted | 4.85 | 70.04 | 89.87 | 1.01 |
| Normal | Real | 5.00 | 30.00 | –90.00 | 1.00 |
| dip slip | Inverted a | 4.66 | 31.76 | 89.85 | 1.15 |
| Inverted b | 4.68 | 31.65 | –90.03 | 0.99 |
| Real | 5.00 | 50.00 | –90.00 | 1.00 |
| Inverted a | 4.41 | 48.43 | –89.92 | 1.11 |
| Inverted b | 4.47 | 48.98 | –89.68 | 1.05 |
| Real | 5.00 | 70.00 | –90.00 | 1.00 |
| Inverted a | 4.59 | 68.25 | –89.90 | 0.73 |
| Inverted b | 4.57 | 68.22 | –90.09 | 1.00 |
| Tensile | Real | 5.00 | 50.00 | – | 1.00 |
| Inverted | 4.10 | 47.35 | – | 0.98 |
| Real | 5.00 | 70.00 | – | 1.00 |
| Inverted | 4.06 | 69.09 | – | 1.10 |
| Real | 5.00 | 90.00 | – | 1.00 |
| Inverted | 3.34 | 90.02 | – | 1.01 |
| Inflation | Real | 5.00 | – | – | 1.00 |
| Inverted | 4.19 | – | – | 0.91 |

Vertical and horizontal ground displacements are used simultaneously.
1The layered parameters are shown in Table 3.
2The layered parameters are shown in Table 4.
3The layered parameters are shown in Table 5.
4The layered parameters are shown in Table 6.
5The layered parameters are shown in the top half of Table 5.
6The layered parameters are shown in the bottom half of Table 5.

In conclusion, in most cases the neglect of the layered
half-space model, especially the low-velocity layers at shal-
lower depth, would cause one to underestimate the depth of
a source. Moreover, using a homogenous half-space to sim-
ulate the coseismic displacements also leads to a misestimate
of the slip and dip angle in the case of tensile and normal
faulting. Only for the reverse dip-slip source with steep dip
angle (>50°) can one neglect the layered half-space model.

| Table 8 | Real and Inverted Results using Layered and Homogenous Half-Space Model for 4 by 6 Subfault Plane |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| Focal Depth | Dip (°) | Rake (°) | Slip (m) |
| (km) | | | |
| Left | Real | 5.00 | 50.00 | 0.00 | 1.00 |
| strike slip | Inverted | 4.40 | 49.48 | 0.02 | 1.07 |
| Real | 5.00 | 70.00 | 0.00 | 1.00 |
| Inverted | 4.50 | 69.82 | –0.01 | 1.06 |
| Real | 5.00 | 90.00 | 0.00 | 1.00 |
| Inverted | 4.37 | 90.04 | –0.06 | 1.04 |
| Reverse | Real | 5.00 | 30.00 | 90.00 | 1.00 |
| dip slip | Inverted | 4.27 | 29.42 | 90.13 | 1.11 |
| Real | 5.00 | 50.00 | 90.00 | 1.00 |
| Inverted | 4.95 | 50.15 | 90.13 | 1.01 |
| Real | 5.00 | 70.00 | 90.00 | 1.00 |
| Inverted | 4.86 | 69.89 | 89.89 | 1.03 |
| Normal | Real | 5.00 | 30.00 | –90.00 | 1.00 |
| dip slip | Inverted a | 4.68 | 31.85 | –90.06 | 1.15 |
| Inverted b | 4.80 | 31.72 | –89.68 | 1.14 |
| Real | 5.00 | 50.00 | –90.00 | 1.00 |
| Inverted a | 4.47 | 48.20 | –89.81 | 1.21 |
| Inverted b | 4.76 | 49.83 | –90.00 | 1.12 |
| Real | 5.00 | 70.00 | –90.00 | 1.00 |
| Inverted a | 4.50 | 69.57 | –89.81 | 0.53 |
| Inverted b | 5.04 | 65.85 | –89.65 | 1.04 |
| Tensile | Real | 5.00 | 50.00 | – | 1.00 |
| Inverted | 3.35 | 43.66 | – | 0.90 |
| Real | 5.00 | 70.00 | – | 1.00 |
| Inverted | 4.07 | 69.15 | – | 1.12 |
| Real | 5.00 | 90.00 | – | 1.00 |
| Inverted | 3.33 | 90.09 | – | 1.01 |
| Inflation | Real | 5.00 | – | – | 1.00 |
| Inverted | 4.19 | – | – | 0.91 |

Horizontal ground displacements alone are considered. The layered pa-
terms are same as table 7.

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References


Appendix

Static Reflection and Transmission Coefficient Matrix

For the P-SV problem, the downgoing and upgoing static reflection and transmission coefficient matrices are defined as

$$R_D = \begin{pmatrix} r_{pp}^+ & r_{ps}^+ \\ r_{sp}^+ & r_{ss}^+ \end{pmatrix}, \quad T_D = \begin{pmatrix} t_{pp}^+ & t_{ps}^+ \\ t_{sp}^+ & t_{ss}^+ \end{pmatrix}$$

$$R_U = \begin{pmatrix} r_{pp}^- & r_{ps}^- \\ r_{sp}^- & r_{ss}^- \end{pmatrix}, \quad T_U = \begin{pmatrix} t_{pp}^- & t_{ps}^- \\ t_{sp}^- & t_{ss}^- \end{pmatrix}$$

(A1)

where

$$r_{pp}^+ = 2\Delta k d_1(\mu_1 - \mu_2)/(\mu_1\Delta_1 + \mu_2),$$

$$r_{sp}^+ = -4\Delta k d_1(\mu_1 + \mu_2\Delta_2)(\mu_1 - \mu_2),$$

$$r_{ps}^+ = (\mu_1 + \mu_2)(\mu_1\Delta_1 - \mu_2\Delta_2)/(\mu_1 + \mu_2\Delta_1 + \mu_2),$$

$$r_{ss}^+ = (\mu_1 + \mu_2)(\mu_1\Delta_1 + \mu_2),$$

(A2)

$$r_{pp}^- = \mu_1(1 + \Delta_1 + \Delta_2 + \Delta_3)(1 + \Delta_2)/(\mu_1\Delta_1 + \mu_2),$$

$$r_{sp}^- = 2\mu_1\Delta_2 d_1(1 + \Delta_1 + \Delta_2 + \Delta_3)/(1 + \Delta_2)/(\mu_1\Delta_1 + \mu_2),$$

$$r_{ps}^- = 0,$$

$$r_{ss}^- = \mu_1(1 + \Delta_1 + \Delta_2 + \Delta_3)/(1 + \Delta_2)/(\mu_1\Delta_1 + \mu_2).$$

(A3)

$$r_{pp}^- = 0,$$

$$r_{sp}^- = (\mu_1\Delta_1 - \mu_2\Delta_2)/(\mu_1\Delta_1 + \mu_2),$$

$$r_{ps}^- = (\mu_1 - \mu_2)/(\mu_1 + \mu_2\Delta_2),$$

$$r_{ss}^- = 0,$$

(A4)

$$r_{pp}^- = \mu_2(1 + \Delta_2)/(\mu_1 + \mu_2\Delta_2),$$

$$r_{sp}^- = 2\Delta_1 d_1\mu_1(1 + \Delta_2)/(\mu_1\Delta_1 + \mu_2),$$

$$r_{ps}^- = 0,$$

$$r_{ss}^- = \mu_2(1 + \Delta_2)/(\mu_1\Delta_1 + \mu_2).$$

(A5)
Here subscripts 1 and 2 denote parameters in upper and lower media, \(d\) is the thickness of the upper medium, and \(k\) is the wavenumber.

For the \(SH\) problem, the static reflection and transmission coefficients are

\[
R_{DL} = (\mu_1 - \mu_2)/(\mu_1 + \mu_2),
\]
\[
T_{DL} = 2\mu_1/(\mu_1 + \mu_2),
\]
\[
R_{UL} = (\mu_2 - \mu_1)/(\mu_1 + \mu_2),
\]
\[
T_{UL} = 2\mu_2/(\mu_1 + \mu_2).
\]

Static Reflection and Transmission Coefficient Matrix with Amplitude Decay

For the elastic problem, the generalized reflection and transmission coefficient matrix at the tops of two neighboring layers is defined as reflection and transmission coefficient matrix with phase delay. Although there is no concept of phase in a static problem, the existence of a layer with a width of \(d_1\) still distorts static displacement fields on adjacent layers. We use the amplitude decay to describe the phenomenon. The relationship between the static generalized reflection and transmission coefficient matrix at the tops of two neighboring layers and the static reflection and transmission coefficients at an interface is

\[
\begin{align*}
\tilde{R}_{p} & = \exp(-2kd_1)\tilde{r}_{pp}, \quad \tilde{R}_{s} = \exp(-2kd_1)\tilde{r}_{ps}, \\
\tilde{T}_{p} & = \exp(-2kd_1)\tilde{t}_{pp}, \quad \tilde{T}_{s} = \exp(-2kd_1)\tilde{t}_{ps}.
\end{align*}
\]

\[
\begin{align*}
\tilde{R}_{pp} & = \exp(-kd_1)\tilde{r}_{pp}, \quad \tilde{R}_{pp} = \exp(-kd_1)\tilde{r}_{pp}, \\
\tilde{T}_{pp} & = \exp(-kd_1)\tilde{t}_{pp}, \quad \tilde{T}_{pp} = \exp(-kd_1)\tilde{t}_{pp}.
\end{align*}
\]

\[
\begin{align*}
\tilde{R}_{ps} & = \exp(-kd_1)\tilde{r}_{ps}, \quad \tilde{R}_{ps} = \exp(-kd_1)\tilde{r}_{ps}, \\
\tilde{T}_{ps} & = \exp(-kd_1)\tilde{t}_{ps}, \quad \tilde{T}_{ps} = \exp(-kd_1)\tilde{t}_{ps}.
\end{align*}
\]

\[
\begin{align*}
\tilde{R}_{ss} & = \exp(-kd_1)\tilde{r}_{ss}, \quad \tilde{R}_{ss} = \exp(-kd_1)\tilde{r}_{ss}, \\
\tilde{T}_{ss} & = \exp(-kd_1)\tilde{t}_{ss}, \quad \tilde{T}_{ss} = \exp(-kd_1)\tilde{t}_{ss}.
\end{align*}
\]