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Editors

Computational Methods for Applied Inverse Problems

Preface

This volume contains the chapters based on the lectures given by invited speakers of the international workshop "Computational Methods for Applied Inverse Problems". The workshop was organized under the auspices of the Chinese Academy of Sciences in the Institute of Geology and Geophysics, located in Beijing, the capital of China, and held during July 12–16, 2010. The workshop was sponsored by the National Science Foundation of China (NSFC), China-Russia Cooperative Research Project (NSFC-RFBR), Knowledge Innovation Programs of Chinese Academy of Sciences (CAS) and the Strategic Priority Research Program of the Chinese Academy of Sciences (CAS). The main goal of the workshop is to gather young participants (mostly mathematicians and geophysicists) from China and overseas together to discuss how to solve inverse and ill-posed problems using different solving strategies. Eminent specialists from China, Russia (partially sponsored by the Russian Foundation of Basic Research), USA, India and Norway were invited to present their lectures. Other young scientists also present their recent researches during the conference.

The book covers many directions in the modern theory of inverse and ill-posed problems — mathematical physics, optimal inverse design, inverse scattering, inverse vibration, biomedical imaging, oceanography, seismic imaging and remote sensing; methods including standard regularization, parallel computing for multidimensional problems, Nyström method, numerical differentiation, analytic continuation, perturbation regularization, filtering, optimization and sparse solving methods are fully addressed. This issue attempts to bridge the gap between theoretical studies of ill-posed inverse problems and practical applications. Let us continue our efforts for further progress.

This book will be helpful to researchers and teachers in developing courses on various inverse and ill-posed problems of mathematical physics, geosciences, designing technology, imaging, high performance computing, inverse scattering and vibration, and so on. It could be also beneficial for senior undergraduate students, graduate and Ph.D. students, recent graduates getting practical experience, engineers and researchers who study inverse and ill-posed problems and solve them in practice.

Chinese Academy of Sciences, Beijing *Yanfei Wang*
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Editor's Preface

Inverse problem theory and methods are driven by applied problems in sciences and engineering. Studies on inverse problems represent an exciting research area in recent decades. The special importance of inverse problems is that it is an interdisciplinary subject related with mathematics, physics, chemistry, geoscientific problems, biology, financial and business, life science, computing technology and engineering.

Inverse problems consist in using the results of actual observations to infer the values of the (model) parameters characterizing the system under investigation. Inverse problems are typically ill-posed in the sense that one of the three items "existence, uniqueness or stability" of the solution may be violated. Inverse problems use modeling design and solving methods to provide a better, more accurate, and more efficient simulation for practical problems.

Methodologies for solving inverse problems involve regularization, optimization and statistics. No one particular method solves all inverse problems. This book provides a background of using regularization and optimization techniques to solve practical inverse problems for the readers who do research in computational/applied mathematics, physical chemistry, engineering, geophysics, image processing and remote sensing, etc. In particular, recent advances of inversion theory and solution methods with applications to practical inverse problems are addressed.

This book *Computational Methods for Applied Inverse Problems* will comprise the following scientific fields:

- Historical background and key issues of general inverse problems;
- Recent advances in regularization theory and new solution methods;
- Optimal inverse design and optimization methods;
- Recent advances in inverse scattering;
- Inverse vibration and data processing;
- Modeling and inversion of the geoscientific problems;
- Analytic, algebraic, statistical and computational methods.

The five main parts of the book are preceded with the first part of an introductory chapter. Chapter 1 written by S. I. Kabanikhin presents us a general idea about inverse problems and key theories on solving problems.

The second part of this book is devoted to presenting recent advances in regularization theory and solving methods. Chapter 2 written by D. V. Lukyanenko and A. G. Yagola proposes a parallel computing technique on multidimensional ill-posed problems. In this chapter, example of a practical problem of restoring

magnetization parameters over a ship body using parallelization is considered. Chapter 3 written by M. T. Nair talks about the theoretical issues of Nyström approximation method for ill-posed problems, numerical issues and error estimates. Chapter 4 written by T. Y. Xiao, H. Zhang and L. L. Hao discusses about the regularizing theories on numerical differentiation. In this chapter, different regularization schemes are presented and compared with extensive numerical simulations. Chapter 5 written by C. L. Fu, H. Cheng and Y. J. Ma shows readers the analytic continuation and regularization. Convergence properties and error estimates are included. Chapter 6 written by G. S. Li discusses about the perturbation regularization method for function reconstruction problem. Four cases of coefficient determination of an advection diffusion equation are addressed. Chapter 7 written by L. V. Zotov and V. L. Panteleev presents some filtering methods for ill-posed problems.

The third part is devoted to inverse design problems and optimization. Chapter 8 written by G. S. Dulikravich and I. N. Egorov describes the alloy design methodology. The inverse problem is formulated as a constrained multi-objective optimization problem and solved using a robust evolutionary optimizer of IOSO type. Chapter 9 written by Z. H. Xiang discusses both the optimal sensor placement design and the regularization method. Practical methods based on the well-posedness analysis of parameter identification procedures and the adaptive adjusting of a-priori information are developed. Chapter 10 written by Y. H. Dai introduces a stable optimization method, called the BFGS method. A general convergence result about the BFGS algorithm is obtained.

The fourth part is devoted to the field of inverse scattering. Chapter 11 written by X. D. Liu and B. Zhang presents the uniqueness results in inverse acoustic and electromagnetic obstacle scattering problems. Some interesting open problems are posed at the end of the chapter. Chapter 12 written by G. Bao and P. J. Li addresses a shape reconstruction problem of inverse scattering. A continuation method for the inverse obstacle scattering problem is developed and details about solving issues are established.

The fifth part is devoted to the inverse vibration problems, data processing and some mathematical problems in biomedical imaging. Chapter 13 written by G. M. Kuramshina, I. V. Kochikov and A. V. Stepanova talks about molecular force field calculations. In particular, they discuss how a-priori model assumptions and *ab initio* quantum mechanical calculations are used for constructing regularizing algorithms for the calculation of molecular force fields. Chapter 14 written by J. J. Liu and H. L. Xu discusses about the mathematical models and image reconstruction realizations of magnetic resonance electrical impedance tomography (MREIT), the harmonic B_z algorithm and the integral equation method are presented.

The last part is devoted to the modeling and inversion problems occurred in geophysics, oceanography, and remote sensing. Chapter 15 written by S. I. Kabanikhin and M. A. Shishlenin discusses about iterative regularization method for solving the inverse hyperbolic problems. Chapter 16 written by H. B. Song, X. H. Huang, L. M. Pinheiro, Y. Song, C. Z. Dong and Y. Bai focuses on a new cross discipline between seismology and physical oceanography. Chapter 17 written by L. J. Gelius provides a framework of understanding and analyzing both diffraction-limited imaging as well as super-resolution. Chapter 18 written by Y. F. Wang, Z. H. Li and C. C. Yang makes a short review seismic migration methods and develops a preconditioned regularizing least squares migration method. Chapter 19 written by Y. F. Wang, J. J. Cao, T. Sun and C. C. Yang extends the conception of compressive sensing to seismic wavefileds interpolation, sparse optimization and regularization methods are fully described. Chapter 20 written by H. Yang presents a quantitative model to characterize the reflectance of land surfaces, regularizing and optimizing issues and multistage inversion strategies are discussed.

The special features of the book are that it provides both novel methods for standard and nonstandard regularization and practical applications in science and engineering. Each chapter is written by respective researchers in their research fields. Illustrations and tables are provided for better understanding of their ideas. Scientists, researchers, engineers, as well as graduate students engaged in applied/computational mathematics, engineering, physical chemistry, geophysics, medical science, image processing, computer science and remote sensing, will benefit from this book.

Finally, we hope that this book will stimulate and inspire new research efforts and the intensive exploration of new promising directions.

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